

Analytic Methods for calculation of R. L. GLUCKSTERN Impedance

Background

- Methods have been developed for $\beta = 1$
 - small obstacle at low frequency
 - impedance of pipe and beam
 - resistive wall
 - calculation of polarizability and susceptibility and generalization to finite walls
 - frequency region near pipe cutoff and broad resonance
 - hole in screen within beam pipe
 - high frequency behavior for one hole
 - resonance within hole
 - high frequency behavior for periodic holes
 - causality
 - usefulness of admittance.
- Need to consider $\beta < 1$ for proton machines like the SNS
 - beam impedance
 - resistive wall
 - small obstacle
 - frequency near pipe cut off
 - hole in screen
 - high frequency behavior (?)
 - absence of causality (?)

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Longitudinal Coupling Impedance

Beam in a perfectly conducting pipe

All quantities in the time domain

$$\rho(r, z; \omega) = \frac{Q}{\pi a^2 v} e^{-j\omega z/v}$$

$$J_z(r, z; \omega) = \frac{Q}{\pi a^2} e^{-j\omega z/v}$$

$$Z_{ll}(\omega) = -\frac{1}{Q^2} \int d^3v \bar{E} \cdot \bar{J}^*$$

$$\frac{Z_{ll}(\omega)}{Z_0} = -\frac{jL}{\pi a^2 \omega} \left[1 - 2 F_l(\sigma_a) I_l(\sigma_a) \right]$$

$$\sigma = \omega/v\gamma \quad , \quad F_l(x) = K_l(x) - \frac{K_0(xb)}{I_0(xb)} I_l(x)$$

For $\sigma_a \ll 1$

$$\frac{Z_{ll}(\omega)}{n Z_0} = -\frac{1}{\rho \gamma^2} \left[\ln \frac{b}{a} + \frac{1}{4} + \phi(\sigma b) \right]$$

$$\phi(y) = \ln \frac{2}{y} - C - \frac{K_0(y)}{I_0(y)} , \quad \phi(0) = 0$$

* Can easily include finite wall conductivity

$$p(y) \rightarrow -\frac{y^2}{4} + O(y^4) \text{ for small } y$$

- Original definition of Z_{11} involves $E_z(r=0)$ and leads to

$$\ln \frac{b}{a} + \frac{1}{2}$$

- More useful is average over the transverse distribution of the beam. Leads to

$$\ln \frac{b}{a} + \frac{1}{4} \quad \text{for uniform transverse distribution}$$

$$\text{or } \ln \frac{b}{a} + \frac{1}{n+4} \quad \text{for } p \rightarrow r''$$

- Allows Z_{11} to be expressed in terms of a surface integral at the wall or on the hole surface

- Similar situation applies for Z_1

$$Q^2 Z_{||}(\omega) = - \int d^3v \vec{E} \cdot \vec{J}^*$$

For a hole in the wall, the contribution from the hole can be written as

$$Q^2 Z_{||}(\omega) = \int_{\text{hole}} dS \vec{E}_{\perp} \times \vec{H}_{10}$$

In terms of electric polarizability χ and magnetic susceptibility ψ (for wavelength much smaller than the hole dimensions)

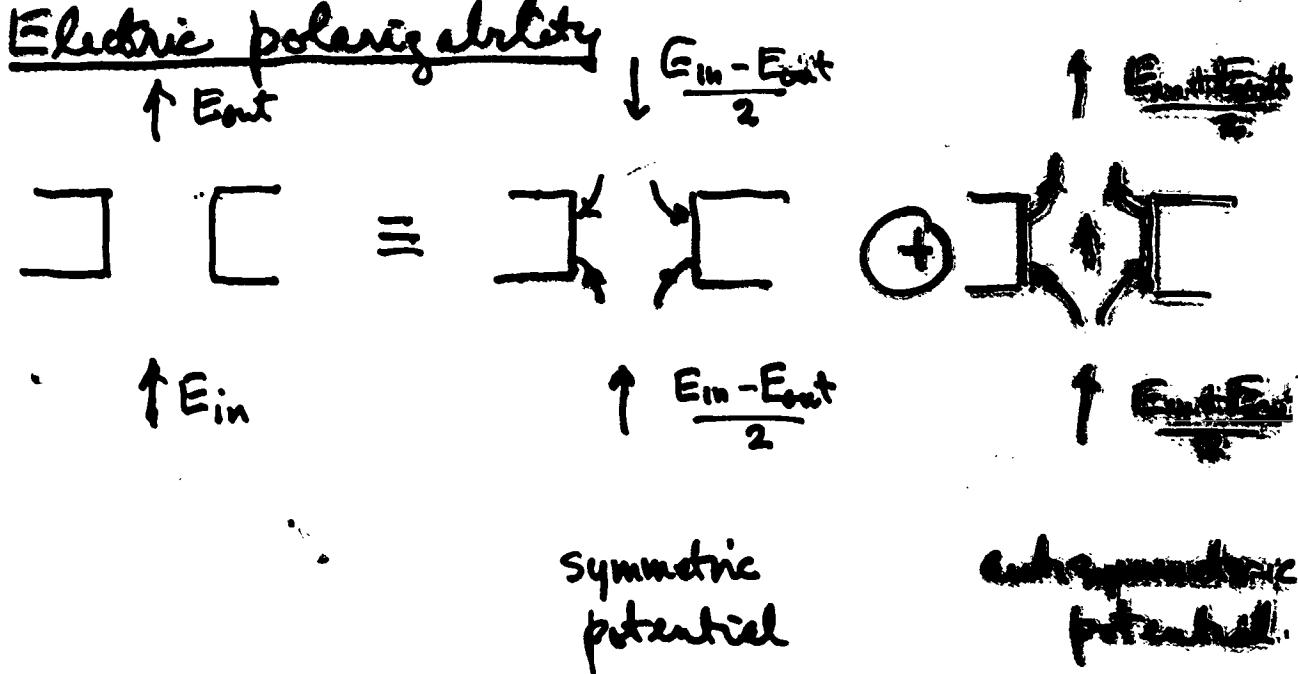
$$Q^2 Z_{||}(\omega) = j \frac{\omega \mu \psi}{2} |H_0|^2 - j \frac{\omega \epsilon \chi}{2} |E_0|^2$$

$$Z_0 H_0 = \beta E_0 = \frac{Q Z_0}{2\pi b} \frac{e^{-j\omega t/v}}{I_0(\sigma b)}$$

$$\frac{Z_{||}(\omega)}{Z_0} = \frac{j \omega}{8\pi^2 b v} \frac{(\beta\psi - \chi/\beta)}{[I_0(\sigma b)]^2}$$

- Impedance changes sign as β decreases from 1
- Reduced cancellation for long narrow slot

Electric polarizability



Solve the electrostatic problem in each case.

$$\chi_{in} = \chi_s + \chi_a, \quad \chi_{out} = \chi_s - \chi_a$$

χ_{in} related to electric dipole seen "inside"

χ_{out} " " " " " " " outside"

- ⊗ Similar decomposition is made for the magnetostatic problem, leading to the "inside" and "outside" magnetic susceptibilities (two components)

Transverse Coupling Impedance / Small hole

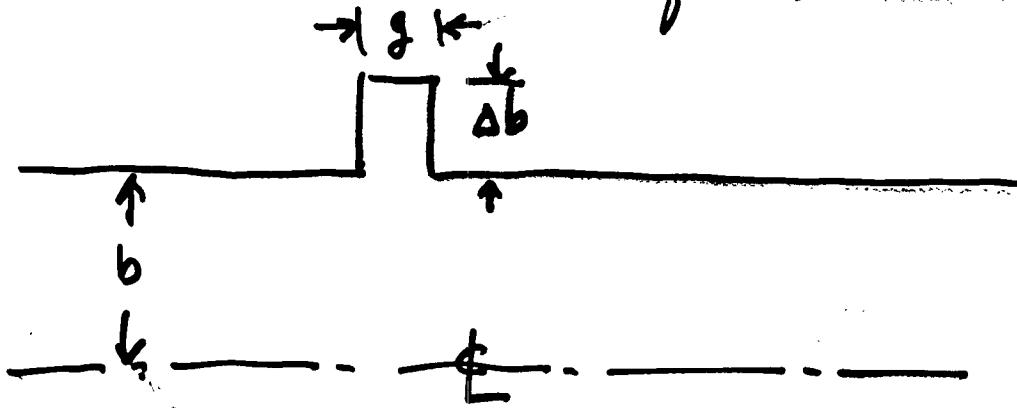
Consider beam to be offset from axis

$$\frac{Z_x(\omega)}{Z_0} = \frac{j \cos^2 \theta}{2\pi^2 b^4} \left[\frac{\sigma b}{2\Sigma_1(\sigma b)} \right]^2 \frac{(\rho^2 \psi - 1)}{\rho}$$

θ is the angle between the direction of beam offset and the plane of the transverse coupling impedance

- ⊗ As β decreases below 1, the sign of the coupling reactance changes, as for the longitudinal impedance

Longitudinal Impedance of a Thin Cavity



For $\beta = 1$, $k = \omega/c$

$$Z_0 Y_{11}(k) = 2\pi k b \left[-\frac{j}{k g \Delta b} + j \frac{2 \ln 2}{\pi} + \sum_{s=1}^{\infty} \frac{e^{-j b_s}}{b_s} \right]$$

$$b_s^2 = k^2 b^2 - p_s^2, \quad J_0(p_s) = 0$$

$$\left[b_s = -j \sqrt{p_s^2 - k^2 b^2} \quad \text{for } p_s > k b \right]$$

- ① $Y_{11}(k)$ dominated by inductive effect of cavity for small kb
- ② Additional contribution from pipe as kb grows
- ③ Real part of impedance when $kb > p_c$ (cutoff)
- ④ Explanation of "broad resonance"

- Consider a screen (with pump holes) inside a beam pipe. For one hole

$$Z_{\parallel} = Z_{\parallel}^{(\text{el})} + Z_{\parallel}^{(\text{mag})}$$

$$Z_0 Y_{\parallel}^{(\text{el})}(k) = j \frac{8\pi^2 b^2}{k} \left[\frac{1}{\chi} - \tilde{W} \right]$$

$$Z_0 Y_{\parallel}^{(\text{mag})}(k) = -j \frac{8\pi^2 b^2}{k} \left[\frac{1}{\psi} + \tilde{V} \right]$$

For $\beta \neq 1$, one can show

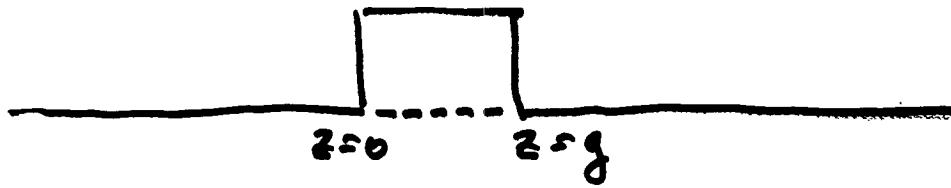
$$\frac{Z_{\parallel}(\omega)}{Z_0} = \frac{j\omega}{8\pi^2 b^2 v [I_0(\omega)]^2} \left[\frac{\beta}{\frac{1}{\psi} + \tilde{V}} - \frac{1}{\beta \left(\frac{1}{\chi} - \tilde{W} \right)} \right]$$

- \tilde{V} and \tilde{W} are complex quantities depending primarily on the pipe geometry. Thus far we have calculated their behavior for a thin screen at relatively low frequency.

- \tilde{V} contains the TM modes which propagate in the waveguide and in the coax between the screen and beam pipe

- \tilde{W} contains the TE modes which propagate in the waveguide and in the coax between the screen and the beam pipe

Integral equation for the longitudinal impedance



$$\int_0^g dz' F(z') \left[K_p(|z'-z|) + K_c(z, z') \right] = -j e^{-jkz}$$

$$\frac{Z_{ll}(\omega)}{Z_0} = \frac{1}{k b [I_0(\omega b)]^2} \int_0^g dz' e^{jkz'/b} F(z')$$

$$K_p(u) = \frac{2\pi j}{b} \sum_{s=1}^{\infty} \frac{e^{-j k b s u / b}}{b_s}$$

$$K_c(z, z') = 4\pi^2 \sum_s \frac{h_e(z) h_e(z')}{k^2 - k_s^2}$$

- At high frequency one needs to evaluate the smoothed parts of the kernels
- For $\beta=1$ there is cancellation between $\frac{w}{v}$ and k for small ϵ
- For $\beta \neq 1$ this cancellation is mitigated

Observations

- Easy to modify small obstacle results for $\beta < 1$
- Departure from small obstacle behavior when λ is comparable to obstacle dimensions
- Resonant effects do not appear to be important
- Need to use ψ_{in} and χ_{in} . Important dependence on wall thickness
- Major modification of small hole results when frequency is above pipe cut-off
- High frequency results change when $\beta < 1$.
~~Causality no longer applies~~ Causality now implies movement of singularities in the complex plane
- Need to look at periodic obstacles for $\beta < 1$
- Admittance is frequently simpler than impedance. Leads to separation into a part related to the obstacle and a part related to the pipe